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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2951**

Unique Paper Code : **42357501**

Name of the Paper : **Differential Equations, CBCS (LOCF)**

Name of the Course : **B.Sc. (Prog.) Physical Sciences/Mathematical Sciences**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) Find the value of A such that the following equation is exact and then solve it. 6

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0.$$

(*b*) Solve the initial-value problem : 6

$$x^2\frac{dy}{dx} + xy = \frac{y^3}{x}, \; y(1) = 1$$

(*c*) Solve the differential equation : 6

$$(x + 2y - 1)dx - (x + 2y + 1)dy = 0.$$

2. (*a*) Find the orthogonal trajectories of the family of curves $y^2 = cx$. 6.5

(*b*) Show that x and $\frac{1}{x^2}$ are linearly independent solutions of the differential equation : 6.5

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$$ on the interval $0 < x < \infty$.

Find the solution of this differential equation which satisfies the conditions

$$y(2)=3,\ y'(2)=-1$$ 6.5

(c) Given that $y = x^2$ is a solution of

$$(x^3-x^2)\frac{d^2y}{dx^2}-(x^3+2x^2-2x)\frac{dy}{dx}+(2x^2+2x-2)y=0.$$

Find a linearly independent solution by reducing the order. Also write the general solution. 6

3. (a) Find the particular solution of the given differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0,\ y(0)=5,\ y'(0)=-3$$

(b) Find a second-order ODE whose fundamental solutions are e^{6x}, e^{-4x}. 6

(c) Solve the differential equation

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-6y=10x^2,\ y(1)=1,\ y'(1)=-6$$ 6

4. (a) Solve the given differential equation using the variation of parameters method $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=4e^{2x}$ 6.5

(b) Find a homogeneous linear ODE of second order for which the function e^{-2x} and xe^{-2x} are solutions. Also show that these solutions are linearly independent. 6.5

(c) Solve the simultaneous differential equation :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$ 6.5

5. (a) Form partial differential equations by 6

(i) eliminating the arbitrary constants 'a' and 'b' from the integral surface

$$(x - a)^2 + (y - b)^2 + z^2 = r^2.$$

(ii) eliminating the arbitrary function f from the integral surface

$$z = xy + f(x^2 + y^2).$$

(b) Find the solution of the Cauchy problem : $xu_x + yu_y = xe^{-u}$, with $u = 0$ when $y = x^2$. 6

(c) Reduce the equation to canonical form and obtain the general solution : 6

$$u_x - yu_y = u + 1.$$

6. (a) Find the general solution of the linear partial differential equation : 6.5

$$x^2u_x + y^2u_y + z(x + y)u_z = 0.$$

(b) Find the solution of the following partial differential equation by the method of separation of variables : 6.5

$$u_x + 5u_y = 0, \; u(x, 0) = 3e^{-2x}$$

(c) Solve the following initial value problem using method of characteristics : 6.5

$u_t + 2u\, u_x = v - x$, $v_t - c\, v_x = 0$ with initial conditions,

$u(x, 0) = x$, $v(x, 0) = x$.